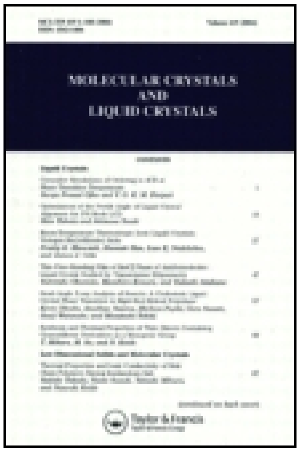

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DYNAMIC HOLOGRAPHIC GRATINGS IN NEMATIC CELL WITH PERIODIC BOUNDARY CONDITIONS

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Abstract Nematic cell with light intensity spatial grating is considered. The influence of periodic spatial modulation of the director anchoring energy and the easy orientation axis at the cell surfaces on the director spatial distribution in the liquid crystal bulk under the action of light is investigated. It is shown that in both cases the director distribution takes the form of a superposition of the cosine gratings with periods which are the linear combinations of the light intensity and boundary condition periods. The Frederiks transition threshold value is calculated for homeotropic cell with periodic modulation of the anchoring energy.

INTRODUCTION

It is well known that nematic liquid crystals (LCs) possess giant optical nonlinear susceptibility which gives the opportunity to record dynamic holographic diffraction gratings[1]. From another side the interest in anchoring transitions, the easy axes governing, LCs with non-homogeneous director distribution is growing because of their extreme importance for the applications[2,3]. The last effects appear due to the transformations of the boundary conditions in time and space.

In our paper we consider the influence of the periodic spatial change of both the easy axis orientation at the cell surfaces and the director anchoring energy on the recording of the dynamic director gratings in the LC bulk by the incident light field with spatially modulated intensity. For the threshold director reorientation it is obtained the dependence of the Frederiks transition threshold value on the spatial change of the director anchoring energy.

SPATIALLY MODULATED EASY ORIENTATION AXIS

The system under consideration is a nematic LC of thickness L restricted with two planes at $z = 0, L$. Two plane monochromatic light waves with equal amplitudes and polarization along the same direction in the XOZ plane are incident on the cell making in consequence of interference in the cell volume the light field in the form

$$\vec{E}^0(\vec{r},t) = \tfrac{1}{2}\left[\vec{E}^0(\vec{r})\exp(-i\omega t) + \vec{E}^{0*}(\vec{r})\exp(i\omega t)\right] \tag{1}$$
$$\vec{E}^0(\vec{r}) = \vec{E}_0\cos\left(\tfrac{1}{2}\Delta q y\right)\exp\left[i(q_x x + q_z z)\right], \qquad \vec{E}_0 = E_0(\cos\beta, 0, -\sin\beta)$$

namely, the light wave intensity grating is recording with the spatial period $T_E = 2\pi/\Delta q$ along the OY axis.

Consider the non threshold director reorientation under the action of the light field (1) in the case of periodic modulation of the director easy orientation axis on the cell surfaces along the axis OY putting here for the sake of simplicity the director anchoring energy with the cell surfaces $W = \infty$.

Planar initial alignment of director.

Let the light field (1) propagates along OZ axis ($q_x = 0, \beta = 0$). Presenting director as $\vec{n} = (\sin\varphi(y,z), \cos\varphi(y,z), 0)$ and minimizing the nematic cell free energy under the one-elastic-constant approximation one can obtain the equation for the director angle $\varphi(y,z)$ and the boundary conditions in the form

$$\Delta\varphi + \frac{\varepsilon_a}{2\pi K}E_0^2\cos^2\left(\tfrac{1}{2}\Delta q y\right)\sin\left[2\left(\varphi_L + \Delta\varphi\sin\frac{y}{\lambda}\right)\right] = 0 \tag{2}$$

$$\varphi\big|_{z=0,L} = \varphi_{0,L} + \Delta\varphi\sin\frac{y}{\lambda} \tag{3}$$

Here $\varepsilon_a = \varepsilon_{\parallel} - \varepsilon_{\perp}$ is the anisotropy of the dielectric susceptibility, K is the Frank's elastic constant, $2\pi\lambda$ is the spatial period of the easy axis modulation. To obtain the equation (2) we assume the polarization vectors of the ordinary and extraordinary waves follow the local orientation of director which corresponds to Mauguin limit[4]. Solving (2) one can obtain the final expression for the director deviation angle (we assume that $\Delta\varphi \ll 1$)

$$\varphi^0(y,z) = \left(1-\frac{z}{L}\right)\varphi_0 + \frac{z}{L}\varphi_L + \frac{\Delta\varphi\left(\sinh\frac{z}{\lambda} + \sinh\frac{L-z}{\lambda}\right)}{\sinh(L/\lambda)}\sin\frac{y}{\lambda} + I(\varphi^1 + \varphi^2) \tag{4}$$

$$\varphi^1 = \sin(2\varphi_L)\left[Z(z,\tfrac{1}{\Delta q})\cos(\Delta q y) + \frac{z}{4L}\left(1-\frac{z}{L}\right)\right] \tag{5}$$

$$\varphi^2 = \Delta\varphi \frac{\cos(2\varphi_L)}{2}\left(Z(z,\lambda_1)\sin\frac{y}{\lambda_1} + Z(z,\lambda_2)\sin\frac{y}{\lambda_2} + 2Z(z,\lambda)\sin\frac{y}{\lambda}\right).$$ Here

$$Z(z,\lambda) = (\lambda/L)^2 \frac{\sinh(z/2\lambda)\sinh((L-z)/2\lambda)}{\cosh(L/2\lambda)},\ I = \frac{\varepsilon_a E_0^2 L^2}{2\pi K},\ \frac{1}{\lambda_1} = \frac{1}{\lambda} + \Delta q,\ \frac{1}{\lambda_2} = \frac{1}{\lambda} - \Delta q$$

One can see that the initial director distribution with the spatial period $T = 2\pi\lambda$ is modified in the light wave field and the additional dynamic gratings with periods $T_E = 2\pi/\Delta q$, $T_1 = 2\pi/(\lambda^{-1} + \Delta q)$, $T_2 = 2\pi/(\lambda^{-1} - \Delta q)$ appear in the LC bulk.

Homeotropic initial alignment of director.

In the homeotropic case we need the solution of Maxwell equations for the field in the cell. Assuming the director deviations from the homeotropic alignment are small and the variation of the light field is smooth at the scale $(\Delta q \varepsilon_a)^{-1}$ we obtain from the linearized by n_x, n_y Maxwell equations the next solution

$$E_x = E_x^0 - \frac{\varepsilon_a}{\varepsilon_\perp} n_x E_z^0,\quad E_y = -\frac{\varepsilon_a}{\varepsilon_\perp} n_y E_z^0,\quad E_z = E_z^0 - \frac{\varepsilon_a}{\varepsilon_{||}} n_x E_x^0 \tag{6}$$

The variation of the nematic cell free energy and further substitution of the field (6) gives the equation for director with the boundary conditions similar to the planar ones (3). Neglecting the terms squared in I we can get the solution in the form

$$n_x = I\frac{\sin 2\beta}{4}\left[\frac{z}{2L}\left(1 - \frac{z}{L}\right) + Z\left(z, \frac{1}{\Delta q}\right)\cos(\Delta q y)\right] \tag{7}$$

$$n_y = n_y^0 + I\Delta\theta \frac{\varepsilon_{||}}{4\varepsilon_\perp}\sin^2\beta\left[R(z,\lambda)\sin\frac{y}{\lambda} + P(z,\lambda_1)\sin\frac{y}{\lambda_1} + P(z,\lambda_2)\sin\frac{y}{\lambda_2}\right]$$

where the functions $R(z,\lambda)$, $P(z,\lambda)$ are too cumbersome in order to present here.

It is seen that the periodic distortion of the n_x director component appears in the LC bulk with the same spatial period as that of the incident field intensity. At the same time the initial modulation of the n_y component is modified in the presence of the field (1) and the diffraction gratings with the spatial periods $T_1 = 2\pi/(\lambda^{-1} + \Delta q)$, $T_2 = 2\pi/(\lambda^{-1} - \Delta q)$ appear in the LC bulk as well.

SPATIALLY MODULATED ANCHORING ENERGY

Let we have the homeotropically aligned nematic with the initial orientation of director along the axis OZ and the LC surface free energy in the form[5]

$$F_S = -\tfrac{1}{2}\int W(y)(\vec{n}\vec{e})^2 dS \tag{8}$$

where $\vec{e}$ is an easy orientation vector on the cell surfaces, $W(y) = W + \alpha(y)$ is the director anchoring energy which depends on the coordinate y, $W > 0$, $|\alpha(y)| < W$

It is convenient to present the director in the nematic cell under the action of the light field (1) in the form $\vec{n} = \{\sin\varphi(y,z), 0, \cos\varphi(y,z)\}$. Then after ordinary variational procedure one can obtain the following equation and boundary conditions for the angle $\varphi(y,z)$

$$\hat{L}\varphi(y,z) = \frac{\varepsilon_a\varepsilon_\perp}{4\pi K\varepsilon_{||}}\sin 2\beta E_0^2\cos^2\left(\tfrac{1}{2}\Delta qy\right), \qquad \hat{L} = \Delta + \frac{\varepsilon_a\varepsilon_\perp E_0^2}{2\pi K\varepsilon_{||}}\cos 2\beta\cos^2\left(\tfrac{1}{2}\Delta qy\right) \tag{9}$$

$$\left(\mp K\frac{\partial\varphi}{\partial z} + W(y)\varphi\right)\Big|_{z=0,L} = 0 \tag{10}$$

Non threshold director reorientation

Consider the case $|\alpha(y)| << W$. One can expand the solution to equation (9) into the eigenfunctions of the operator $\hat{L}$ and then satisfy the boundary conditions (10). As a result the solution takes the form

$$\varphi(y,z) = \sum_{\lambda}\sum_{m=0}^{\infty} C_m^{\lambda}\cos(m\Delta qy)\left(\sin\lambda_m z + \frac{K\lambda_m}{W_m^{\lambda}}\cos\lambda_m z\right) \tag{11}$$

where λ_m are the solutions to the equation

$$\tan\lambda L = \frac{2W_m^{\lambda}K\lambda}{(K\lambda)^2 - (W_m^{\lambda})^2} \tag{12}$$

Here $W_m^{\lambda} = W + \sum_{l=0}^{\infty}\alpha_{ml}\frac{C_l^{0,\lambda}}{C_m^{0,\lambda}}$, $\alpha_{ml} = \frac{1}{T}\left(1 - \tfrac{1}{2}\delta_{m,0}\right)\int_T^T \alpha(y)\cos(m\Delta qy)\cos(l\Delta qy)dy$, T is the spatial period of the problem, $C_l^{0,\lambda}$ are the known coefficients in the solution (11) for the case $\alpha(y)=0$. On substitution of the series (11) into the equation (9) one obtains the infinite system of equations for the coefficients C_m^λ. It can be proved that the ratio $C_m^\lambda / C_{m+1}^\lambda = O(1/m^2)$, thus the series (11) converges absolutely and uniformly. It makes possible to restrict the consideration to the finite number of equations for C_m^λ. The diffraction efficiencies of the director dynamic gratings generated in nematic are proportional to the parameters $\eta_m = C_m^\lambda / E_0^2$. If we restrict our consideration, for example, to the terms with $m \leq 2$ the director distribution along the OY axis is approximately a superposition of the cosine gratings with periods $T_E - 2\pi/\Delta q$, $T_E^1 = \pi/\Delta q$. Putting $\alpha(y) = \alpha\cos(k\Delta q y)$, $\beta = \pi/8$ and $E_0 << E_0^{th}$, where E_0^{th} is the Frederiks transition threshold value one can obtain for the grating with period T_E and the most intensive director mode in (11) with $\lambda = \lambda_{\min}$ the analytical expression for the diffraction parameter η_1. From this follows that the relative change of the η_1 due to the periodic modulation of the anchoring energy is

$$\chi_1 = \frac{\eta_1 - \eta_1^0}{\eta_1^0} = -4\frac{\alpha K}{W^2 L}, \qquad \text{if } k = 1 \tag{13}$$

$$\chi_1 = -2\frac{\alpha K}{W^2 L}\frac{T_E^2}{T_E^2 + 4L^2}, \qquad \text{if } k = 2 \tag{14}$$

where η_1^0 is the diffraction parameter in the absence of the anchoring energy modulation ($\alpha = 0$). It is seen that the diffraction efficiency of the grating depends on the sign of the quantity α (the phase shift between the modulation waves of the incident light intensity and the director anchoring energy). If $k \geq 3$ the value χ_1 is considerably smaller than in the cases of $k = 1, 2$; and if $k = 0.5$ we get $\chi_1 = 0$.

Frederiks transition threshold value.

Putting $\beta = 0$ in (1) we can consider the threshold director reorientation and find the threshold value E_0^{th} for the arbitrary smooth function $\alpha(y)$. The equation (9) is now homogeneous. Seeking its solution in the form (11) we obtain the infinite system of homogeneous equations for C_m^λ. One can restrict consideration to the finite number of equations with any desired accuracy. Then the condition of the non-trivial solution to this system gives the equation for the E_0^{th}.

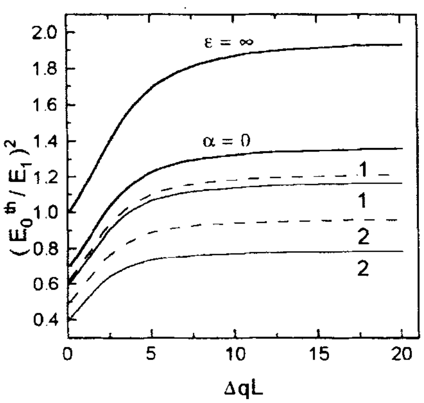

FIGURE 1 $k = 1$ (solid), $k = 2$ (dashed) $\tilde{\alpha} = 4$ (1), 8 (2). $\tilde{\alpha} = \alpha L/K$

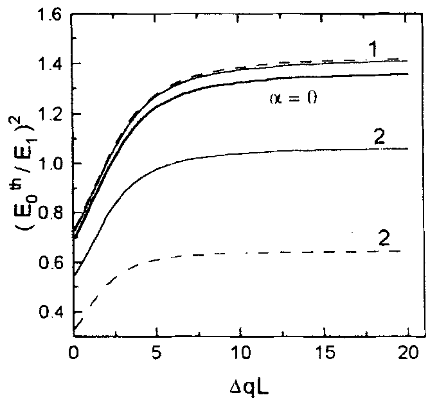

FIGURE 2 $k = 1$ (solid), $k = 0.5$ (dashed) $\tilde{\alpha} = 8$ (1), - 8 (2). $\tilde{\alpha} = \alpha L/K$

The numerical solution to this equation for the $\alpha(y) = \alpha\cos(k\Delta qy + \gamma)$ and $k = 1,2$ (see Fig. 1) shows that E_0^{th} reaches the minimum at $k = 1$ and approaches $E_0^{th}(\alpha = 0)$ with the increasing of integer k. We should note that the threshold value does not depend on the phase shift γ. Besides $E_0^{th} = E_0^{th}(\alpha = 0)$ in the case of $k = 1/n$ where the integer $n \geq 2$. The dependence of E_0^{th} on the ΔqL for the $\alpha(y) = \alpha|\cos(k\Delta qy)|$ and $k = 0.5, 1$ is presented in Fig.2. As one would expect, the threshold value increases with α. For $\alpha < 0$ the minimum of E_0^{th} is gained at $k = 0.5$ and for $\alpha > 0$ E_0^{th} approaches $E_0^{th}(\alpha = 0)$ with the increasing of integer k.

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